Problem 13.3

A 200 kg and a 500 kg object are separated by 4.00 meters.

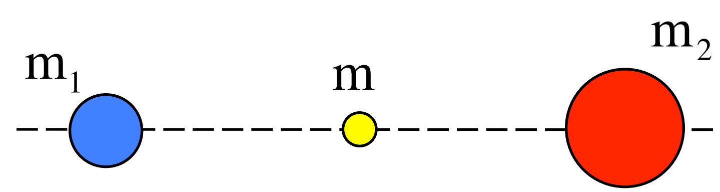
a.) How large is the gravitational force exerted on a 50.0 kg object located between the two?

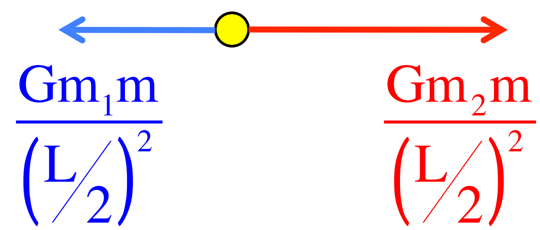
We are about to use Newton's universal gravitational force function. Formally that force function is stated as:

$$\vec{F} = \left(\frac{Gm_1m}{r^2}\right)(-\hat{r})$$

where m_1 is the field-producing mass and m the field-experiencing mass, *r* is the distance between the *center of mass* of each body and the universal gravitational constant $G = 6.67x10^{-11}\ N \bullet m^2/kg^2$. What is slightly dicey about this relationship is that it is not presented in Cartesian coordinates but in polar spherical coordinates. That is, the unit vector, $-\hat{r}$, signifies a force vector that is in the *minus radial direction,* relative to the field producing mass (that is, it points *toward* the field producing mass). That means that this expression is best used to determine the MAGNITUDE of the gravitational force with your manually putting in the appropriate sign of the force on the body at the instant of interest.

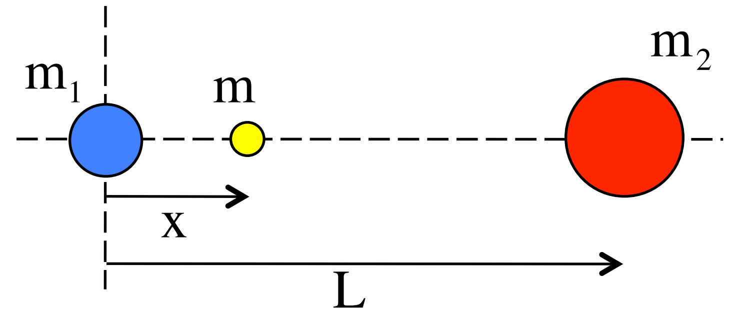
For this problem, the sketch to the right identifies the players. We are going to sum the forces., so a f.b.d. on the mass "m" will help. That is shown to the right. With it, we can write:



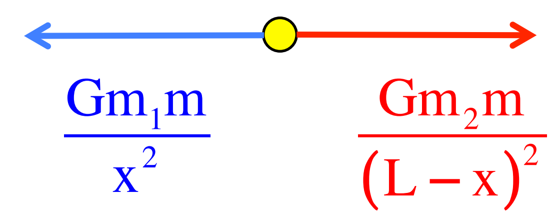


$$
\begin{aligned}
\vec{F}_{net} &= \vec{F}_1 + \vec{F}_2 \\
&= -\left(\frac{Gm_1m}{\left(L/2\right)^2}\right) + \left(\frac{Gm_2m}{\left(L/2\right)^2}\right) \\
&= \left(\frac{Gm}{\left(L/2\right)^2}\right)\left(-m_1 + m_2\right) \\
&= \left(\frac{\left(6.67\text{x}10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(50.0\ \text{kg}\right)}{\left(2.00\ \text{m}\right)^2}\right)\left(-\left(200.\ \text{kg}\right) + \left(500.\ \text{kg}\right)\right) \\
&= 2.50\text{x}10^{-7}\ \text{N}
\end{aligned}
$$

b.) Is there a place where the net force on "m" will be zero, and if so, where is it?



The answer is "yes," and the position will have to be closer to m_1 than m_2. Using the coordinate axis provided:



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = 0$$

$$\Rightarrow \quad \frac{Gm_1m}{x^2} = \frac{Gm_2m}{(L-x)^2}$$

$$\Rightarrow \quad m_1(L-x)^2 = m_2x^2$$

$$\Rightarrow \quad m_1\left(x^2 - 2Lx + L^2\right) = m_2x^2$$

$$\Rightarrow \quad (200.\ kg)\left(x^2 - 2(4.00\ m)x + (4.00\ m)^2\right) = (500.\ kg)x^2$$

$$\Rightarrow \quad 200x^2 - 1600x + 3200 - 500x^2 = 0$$

$$\Rightarrow \quad -3x^2 - 16x + 32 = 0$$

$$\Rightarrow \quad x = 1.55\ m$$